Bargaining with Neighbors: Is Justice Contagious?*

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Abstract

We investigate evolutionary dynamics for the simplest symmetric bargaining game in two different settings. Bargaining with strangers, modeled by the replicator dynamics, sometimes converges to fair division—sometimes not. Bargaining with neighbors, modeled by imitation dynamics on a spatial lattice, almost always converges to fair division. Furthermore, convergence is remarkably rapid. We vary to model the isolate the reasons for these differences. In various forms of bargaining with neighbors, justice is contagious.

1 Justice

What is justice? The question is harder to answer in some cases than in others. We focus on the easiest case of distributive justice. Two individuals are to decide how to distribute a windfall of a certain amount of money. Neither is especially entitled, or especially needy, or especially anything their positions are entirely symmetric. Their utilities derived from the distribution may, for all intents and purposes, be taken simply as the amount of money received. If they cannot decide, the money remains undistributed and neither gets any. The essence of the situation is captured in the simplest version of a bargaining game devised by John Nash.[1] Each person decides on a bottom line demand. If those demands do not jointly exceed the windfall, then each person gets his demand; if not, no one gets anything. This game is often simply called "divide-the-dollar".

In the ideal simple case, the question of distributive justice can be decided by two principles:

Optimality: A distribution is not just if under an alternative distribution all recipients would be better off.

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[1] Nash, J. (1950) "The Bargaining Problem" *Econometrica* 18: 155–62.

Equity: If the position of the recipients is symmetric, then the distribution should be symmetric. That is to say, it does not vary when we switch the recipients.

Since we stipulate that the position of the two individuals is symmetric, Equity requires that the just distribution must give them the same amount of money. Optimality then rules out such unlikely schemes as giving each one dime and throwing the rest away—each must get half the money.

There is nothing new about our two principles. Equity is the simplest consequence of the theory of distributive justice in Aristotle's *Politics*. It is a consequence of Kant's categorical imperative. Utilitarians tend to stress optimality, but are not completely insensitive to equity. Optimality and equity are the two most uncontroversial requirements in John Nash's axiomatic treatment of bargaining. If you ask people to judge the just distribution their answers show that optimality and equity are powerful operative principles.[2] So, although nothing much hangs on it, we will feel free to use moral language and to call the equal split "fair division" in divide the dollar.

2 Rationality, Behavior, Evolution

Two rational agents play the divide-the-dollar game. Their rationality is common knowledge. What do they do? The answer that game theory gives us is that any combination of demands is compatible with these assumptions. For example, Jack may demand 90% thinking that Jill will only demand 10% on the assumption that Jill thinks that Jack will demand 90% and so forth, while Jill demands 75% thinking that Jack will demand 25% on the assumption that Jack thinks that Jill will demand 75% and so forth. Any pair of demands is rationalizable, in that it can be supported by a hierarchy of conjectures for each player, compatible with common knowledge of rationality. In the example given, these conjectures are quite mistaken.

Suppose we add the assumption that each agent somehow knows what the other will demand. Then any combination of demands that total the whole sum to be divided is still possible. For example, suppose that Jack demands 90% knowing that Jill will demand 10% and Jill demands 10% knowing that Jack will demand 90%. Then each player is maximizing payoff given the demand of the other. That is to say that this is a Nash equilibrium of divide the dollar. If the dollar is infinitely divisible, then there are an infinite number of such equilibria.

If experimental game theorists have people actually play divide-the-dollar, they always split equally.[3] This is not always true in more complicated bargain-

[2] Yaari, M. and Bar-Hillel, M. (1981) "On Dividing Justly" *Social Choice and Welfare* 1: 1–24.

[3] Nydegger, R.V. and Owen, G. (1974) "Two-Person Bargaining, An Experimental Test of the Nash Axioms" *International Journal of Game Theory* 3:239–50; Roth, A. and Malouf, M. (1979) "Game Theoretic Models and the Role of Information in Bargaining" *Psychological Review* 86:574–594; Van Huyck, J., Battalio, R. Mathur, S. Van Huyck, P. and Ortmann, A. "On the Origin of Convention: Evidence From Symmetric Bargaining Games" (1995) *International Journal of Game Theory* 24: 187–212.

ing experiments where there are salient asymmetries, but it is true in divide-the-dollar. Rational choice theory has no explanation of this phenomenon. It appears that the experimental subjects are using norms of justice to select a particular Nash equilibrium of the game. But what account can we give for the existence of these norms?

Evolutionary game theory (reading "evolution" as cultural evolution) promises an explanation, but the promise is only partially fulfilled. Demand $\frac{1}{2}$ is the only evolutionarily stable strategy in divide-the-dollar.[4] It is the only strategy such that if the whole population played that strategy, no small group of innovators, or "mutants", playing a different strategy could achieve an average payoff at least as great as the natives. If we could be sure that this unique evolutionarily stable strategy would always take over the population, the problem would be solved.

But we cannot be sure that this will happen. There are states of the population where some fraction of the population makes one demand and some fraction makes another that are evolutionarily stable. The state where half the population demands $\frac{1}{3}$ and half the population demands $\frac{2}{3}$ is such an evolutionarily stable polymorphism of the population. So is the state where $\frac{2}{3}$ of the population demands 40% and $\frac{1}{3}$ of the population demands 60%. We can think of these as pitfalls along the evolutionary road to justice.

How important are these polymorphisms? To what extent do they compromise the evolutionary explanation of the egalitarian norm? We cannot begin to answer these questions without explicitly modeling the evolutionary dynamics and investigating the size of their basins of attraction.

3 Bargaining with Strangers

The most widely studied dynamic evolutionary model is a model of interactions with strangers. Suppose that individuals are paired at random from a very large population to play the bargaining game. We assume that the probability of meeting a strategy can be taken as the proportion of the population that has that strategy. The population proportions evolve according to the replicator dynamics. The proportion of the population using a strategy in the next generation is the proportion playing that strategy in the current generation mutiplied by a "fitness factor". This fitness factor is just the ratio of the average payoff to this strategy to the average payoff in the whole population.[5] Strategies that do better than average grow; those that do worse than average shrink. This dynamics arose in biology as a model of asexual reproduction, but more to the

[4] Sugden, R. (1986) *The Economics of Rights, Cooperation and Welfare* Oxford: Blackwell.

[5] This is the discrete time version of the replicator dynamics, which is most relevant in comparison to the alternative "bargaining with neighbors" dynamics considered here. There is also a continuous time version. See Hofbauer, J. and K. Sigmund (1988) *The Theory of Evolution and Dynamical Systems* Cambridge: Cambridge University Press; J. Weibull (1997) *Evolutionary Game Theory* Cambridge, Mass.:MIT; Samuelson, L. (1997) *Evolutionary Games and Equilibrium Selection* Cambridge, Mass. as comprehensive references.

point here, it also has a cultural evolutionary interpretation where strategies are imitated in proportion to their success.[6]

The basins of attraction of these polymorphic pitfalls are not negligible. A realistic version of divide-the-dollar will have some finite number of strategies instead of the infinite number that we get from the idealization of infinite divisibility. For a finite number of strategies, the size of a basin of attraction of a population state makes straightforward sense. It can be estimated by computer simulations. We can consider coarse-grained or fine grained versions of divide-the-dollar; we can divide a stack of quarters, or of dimes, or of pennies. Some results of simulations persist across a range of different granularities. Equal division always has the largest basin of attraction and it is always greater than the basins of attractions of all the polymorphic pitfalls combined. If you choose an initial population state at random it is more probable than not that the replicator dynamics will converge to a state of fixation of demand half. Simulation results range between fifty-seven and sixty-three percent of the initial points going to fair division. The next largest basin of attraction is always that closest to the equal split—for example, the 4–6 polymorphism in the case of dividing a stack of ten dimes and the 49–51 polymorphism in the case of dividing a stack of 100 pennies. The rest of the polymorphic equilibria follow the general rule—the closer to fair division, the larger the basin of attraction.

For example, the results running the discrete replicator dynamics to convergence and repeating the process 100,000 times on the game of dividing 10 dimes are given in table 1: The projected evolutionary explanation seems to fall

Fair Division	62,209
4–6 Polymorphism	27,469
3–7 Polymorphism	8,801
2–7 Polymorphism	1,483
1–9 Polymorphism	38
0–10 Polymorphism	0

Table 1: Convergence results for replicator dynamics—10,000 trials

somewhat short. The best we might say on the basis of pure replicator dynamics is that fixation of fair division is more likely that not, and that polymorphisms far from fair division are quite unlikely.

We can say something more if we inject a little bit of probability into the model. Suppose that every once and a while a member of the population just picks a strategy at random and tries it out—perhaps as an experiment, perhaps just as a mistake. Suppose we are at a polymorphic equilibrium, for instance the 4–6 equilibrium in the problem of dividing 10 dimes. If there is some fixed

[6] Bjornerstedt, J. and Weibull, J. (1996) "Nash Equilibrium and Evolution by Imitation". In K. Arrow et. al. (eds.) *The Rational Foundations of Economic Behavior* New York: Macmillan, 155–71; Schlag, K. (1996) "Why Imitate, and if so How?" Discussion Paper B–361. University of Bonn, Bonn, Germany.

probability of an experiment (or mistake), and if experiments are independent, and if we wait long enough, there will be enough experiments of the right kind to kick the population out of the basin of attraction of the 4–6 polymorphism and into the basin of attraction of fair division and the evolutionary dynamics will carry fair division to fixation. Eventually, experiments or mistakes will kick the population out of the basin of attraction of fair division, but we should expect to wait much longer for this to happen. In the long run the system will spend most of its time in the fair division equilibrium. Peyton Young showed that if we take the limit as the probability of someone experimenting gets smaller and smaller, the ratio of time spent in fair division approaches one. In his terminology, fair division is the stochastically stable equilibrium of this bargaining game.[7]

This explanation gets us a probability arbitrarily close to one of finding a fair division equilibrium if we are willing to wait an arbitrarily long time. But one may well be dissatisfied with an explanation that lives at infinity. (Putting the limiting analysis to one side, pick some plausible probability of experimentation or mistake and ask yourself how long you would expect it to take in a population of 10,000, for 1334 demand 6 types to simultaneously try out being demand 5 types and thus kick the population out of the basin of attraction of the 4–6 polymorphism and into the basin of attraction of fair division.)[8] The evolutionary explanation still seems less than compelling.

4 Bargaining with Neighbors

The model of random encounters in an infinite population that motivates the replicator dynamics may not be the right model. Suppose interactions are with neighbors. Some investigations of cellular automaton models of prisoner's dilemma and a few other games show that interactions with neighbors may produce dynamical behavior quite different from that generated by interactions with strangers.[9] Bargaining games with neighbors have not, to the best of our knowledge, previously been studied.

Here we investigate a population of 10,000 arranged on a 100 by 100 square lattice. As the neighbors of an individual in the interior of the lattice we take the eight individuals to the N, NE, E, SE, S, SW, W, NW. This is called the

[7]Foster, D. and Young, P. (1990) "Stochastic Evolutionary Game Dynamics" *Theoretical Population Biology* 38:219–32; Young, P. (1993a) "An Evolutionary Model of Bargaining" *Journal of Economic Theory* 59: 145–68; Young, P. (1993b) "The Evolution of Conventions" *Econometrica* 61: 57–94.

[8]See Ellison, G. (1993) "Learning, Local Interaction and Coordination" *Econometrica* 61:1047–1071 and Axtell ,R. L., Epstein, J. M. and Young, H.P. (1999) "The Emergence of Economic Classes in an Agent-Based Bargaining Model" preprint Brookings Institution, for discussion of expected waiting times.

[9]Pollack, G. B. (1989) "Evolutionary Stability on a Viscous Lattice" *Social Networks* 11:175–212; Nowak, M. and May, R. (1992) "Evolutionary Games and Spatial Chaos" *Nature* 359: 826–29; Lindgren, K. and Nordahl, M. (1994) "Evolutionary Dynamics in Spatial Games" *Physica* D 75: 292–309; Anderlini, L. and Ianni, A. (1997) "Learning on a Torus" in *The Dynamics of Norms* ed C. Bicchieri, Jeffrey, R. and Skyrms. B. Cambridge: Cambridge University Press, 87–107.

Moore(8) neighborhood in the cellular automaton literature.[10] The dynamics is driven by imitation. Individuals imitate the most successful person in the neighborhood. A generation—an iteration of the discrete dynamics—has two stages. First each individual plays the divide-ten-dimes game with each of her neighbors using her current strategy. Summing the payoffs gives her current success level. Then each player looks around her neighborhood and changes her current strategy by imitating her most successful neighbor, providing that her most successful neighbor is more successful than she is. Otherwise she does not switch strategies. (Ties are broken by a coin flip.)

In initial trials of this model fair division always went to fixation. This cannot be a universal law, since you can design "rigged" configurations where a few demand $\frac{1}{2}$ players are, for example, placed in a population of demand 4 and demand 6 players with the latter so arranged that there is a demand 6 type who is the most successful player in the neighborhood of every demand $\frac{1}{2}$ player. Start enough simulations at random starting points and sooner or later you will start at one of these.

We ran a large simulation starting repeatedly at randomly chosen starting points. Fair division went to fixation in more that 99.5% of the trials. The cases where it didn't were all cases where the initial population of 10000 contained fewer than 17 demand $\frac{1}{2}$ players. Furthermore, convergence was remarkably quick. Mean time to fixation of fair division was about 16 generations. This may be compared with a mean time to convergence[11] in discrete replicator dynamics of 46 generations, and with the ultra-long-run character of stochastically stable equilibrium.

It is possible to exclude fair division from the possible initial strategies in the divide-ten-dimes game and start at random starting points that include the rest. If we do this all strategies other than demand 4 dimes and demand 6 dimes are eliminated and the 4–6 polymorphic population falls into a "blinking" cycle of period 2. If we then turn on a little bit of random experimentation or "mutation" allowing the possibility of Demand 5, we find that a soon as a very small clump of Demand 5 players arises, it systematically grows until it takes over the whole population-as illustrated in figure 1. Justice is contagious.[12]

5 Robustness

The bargaining with neighbors model of the last section differs from the bargaining with strangers model in more than one way. Might the difference in behavior that we have just described be due to the imitate-the-most-successful dynamics rather than the neighbor effect? To answer this question we ran simulations varying these factors independently.

[10]We find that behavior is not much different if we use the von Neumann neighborhood: N, S, E, W, or a larger Moore neighborhood.

[11]At .9999 level to keep things comparable.

[12]Ellison (1993) found such contagion effects in local interaction of players arrranged on a circle and playing pure coordination games.

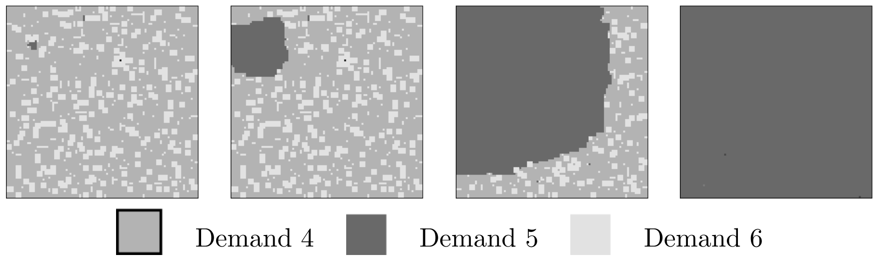

Figure 1: The steady advance of fair division

We consider both fixed and random neighborhoods. The models using fixed neighborhoods use the Moore (8) neighborhood described above. In the alternative random neighborhood model, each generation a new set of "neighbors" is chosen at random from the population for each individual. That is to say, these are neighborhoods of strangers.

We investigated two alternative dynamics. One imitates the most successful neighbor as in out bargaining with neighbors model. The other tempers the all-or-nothing character of Imitate-the-Best. Under it an individual imitates one of the strategies in its neighborhood that is more successful that it (if there are any) with relative probability proportional to their success in the neighborhood. This is a move in the direction of the replicator dynamics. In table 2, A and B are bargaining with neighbors, with imitate-the-best-neighbor and imitate-with-probability-proportional-to-success dynamics respectively. The results are barely distinguishable. C and D are the random neighborhood models corresponding to A and B respectively. These results are much closer to those given for the replicator dynamics in table 1. The dramatic difference in convergence to fair division between our two models is due to the structure of interaction with neighbors.

	Bargaining with Neighbors		Bargaining with Strangers	
	A	B	C	D
0–10	0	0	0	0
1–9	0	0	0	0
2–8	0	0	54	57
3–7	0	0	550	556
4–6	26	26	2560	2418
fair	9972	9973	6833	6964

Table 2: Convergence results for five series of 10,000 trials

6 Analysis

Why is justice contagious? A strategy is contagious if an initial "patch" of that strategy will extend to larger and larger patches. The key to contagion of a strategy is interaction along the edges of the patch, since in the interior the strategy can only imitate itself.[13]

Consider an edge with demand 5 players on one side, and players playing the complementary strategies of one of the polymorphisms on the other. Since the second rank of demand 5 players always meet their own kind, they each get a total payoff of 40 from their eight neighbors. Players in the first rank will therefore imitate them unless a neighbor from the polymorphism gets a higher payoff. The low strategy in a polymorphic pair cannot get a higher payoff. So if demand 5 is to be replaced at all, it must be by the high strategy of one of the polymorphic pairs.

In the 4–6 polymorphism the polymorphism with the greatest basin of attraction in the replicator dynamics this simply cannot happen, even in the most favorable circumstances. Suppose that we have someone playing demand 6 in the first rank of the polymorphism, surrounded on his own side by compatible demand 4 players to boost his payoff to the maximum possible.[14] Since he is in the first rank, he faces three incompatible demand 5 neighbors. He has a total payoff of 30 while his demand 5 neighbors have a total payoff of 35. Demand 5 begins an inexorable march forward as illustrated in table 3. (The pattern is assumed to extend in all directions for the computation of payoffs of players at the periphery of what is shown in the figure.)

Initial					Iteration 1			
5	5	4	4		5	5	5	4
5	5	4	4	$\Longrightarrow$	5	5	5	4
5	5	6	4		5	5	5	4
5	5	4	4		5	5	5	4
5	5	4	4		5	5	5	4

Table 3: Fair division vs. 4–6 polymorphism

If we choose a polymorphism that is more extreme, however, it is possible for the high strategy to replace some demand 5 players for a while. Consider the 1–9 polymorphism, with a front line demand 9 player backed by compatible demand 1 neighbors. The demand 9 player gets a total payoff of 45 more than anyone else and thus is imitated by all his neighbors. This is shown in the first transition in table 4:

[13] For this reason, "frontier advantage" is used to define an unbeatable strategy in Eshel, E., Sansone, E., and Shaked, A. (1996) "Evolutionary Dynamics of Populations with a Local Interaction Structure" working paper, University of Bonn.

[14] In situating the high strategy of the polymorphic pair in a sea of low strategy players, we are creating the best case scenario for the advancement of the polymorphism into the patch of demand 5 players.

Initial		Iteration 1		Iteration 2		Iteration 3
5 5 1 1 1		5 5 5 1 1		5 5 5 5 1		5 5 5 5 5
5 5 1 1 1		5 5 5 1 1		5 5 5 5 9		5 5 5 5 9
5 5 1 1 1		5 9 9 9 1		5 5 9 9 9		5 5 5 9 9
5 5 9 1 1	$\Longrightarrow$	5 9 9 9 1	$\Longrightarrow$	5 5 9 9 9	$\Longrightarrow$	5 5 5 9 9
5 5 1 1 1		5 9 9 9 1		5 5 9 9 9		5 5 5 9 9
5 5 1 1 1		5 5 5 1 1		5 5 5 5 9		5 5 5 5 9
5 5 1 1 1		5 5 5 1 1		5 5 5 5 1		5 5 5 5 5

Table 4: Fair division vs. 1–9 polymorphism

But the success of the demand 9 strategy is its own undoing. In a cluster of demand 9 strategies it meets itself too often and does not do so well. In the second transition demand 5 has more than regained its lost territory, and in the third transition it has solidly advanced into 1–9 territory.

Analysis of the interaction along an edge between demand 5 and other polymorphisms is similar to one of the cases analyzed here.[15] Either the polymorphism cannot advance at all, or the advance creates the conditions for its immediate reversal. A complete analysis of this complex system is something that we cannot offer. But the foregoing does offer some analytic insight into the contagious dynamics of equal division in Bargaining with Neighbors.

7 Conclusion

Sometimes we bargain with neighbors, sometimes with strangers. The dynamics of the two sorts of interaction are quite different. In the bargaining game considered here, bargaining with strangers modeled by the replicator dynamics leads to fair division from a randomly chosen starting point about 60% of the time. Fair division becomes the unique answer in bargaining with strangers if we change the question to that of stochastic stability in the ultra long run. But long expected waiting times call the explanatory significance of the stochastic stability result into question.

Bargaining with neighbors almost always converges to fair division and convergence is remarkably rapid. In bargaining with neighbors, the local interaction generates clusters of those strategies that are locally successful. Clustering and local interaction together produce positive correlation between like strategies. As noted elsewhere[16], positive correlation favors fair division over the polymorphisms. In bargaining with neighbors, this positive correlation is not something externally imposed but rather an unavoidable consequence of the dynamics of local interaction. As a consequence, once a small group demand $\frac{1}{2}$ players is formed, justice becomes contagious and rapidly takes over the entire population.

[15] With some minor complications involving ties.

[16] Skyrms, B (1994) "Sex and Justice" *The Journal of Philosophy* 91:305–20 and Skyrms, B. (1996) *Evolution of the Social Contract* N.Y.: Cambridge University Press.

Both bargaining with strangers and bargaining with neighbors are artificial abstractions. In initial phases of human cultural evolution bargaining with neighbors may be a closer approximation to the actual situation than bargaining with strangers. The dynamics of bargaining with neighbors strengthens the evolutionary explanation of the norm of fair division.