On the Redress of Grievances

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Consider a probabilistic decision procedure which awards a status of *winning* or *losing* to one of two people. (E.g., assigning a kidney for transplant to one of two people.) The decision procedure is *fair* if both people have an equal chance of winning. The decision procedure is *unfair* if one person has chance of winning greater than 50%. In an unfair decision procedure, I shall call the person with a chance of winning less than 50% the *aggrieved* and the person with a chance of winning greater than 50% the *favoured*, independent of whom actually wins or loses.

In order to defend this use of terminology, I assume that both parties want to participate in a fair decision procedure and that neither party had a hand in designing the particular procedure in use. Furthermore, if the procedure is, in fact, unfair, neither the aggrieved nor the favoured know this at the outset. This means that, if a procedure is, in fact, unfair, it is so as a result of choices made by a third party, either deliberately (in the sense of trying to help the favoured) or accidentally (in the sense of an error in design). And, finally, let us assume that an unfair decision procedure is such that it is not the case that the person actually favoured could have been the aggrieved.[1]

[1]This last requirement is needed because an unfair decision procedure can be converted into a fair decision procedure by introducing an element of randomness into the assignment of persons to roles. Suppose we have a biased coin with the probability of heads being $p > \frac{1}{2}$, and we will flip the biased coin to decide whether person 1 or person 2 wins. How do we choose whether person 1 is assigned to heads or tails? If we flip a *fair* coin to do this, then the probability of person 1 winning is:

$$\begin{aligned}\Pr(\text{Win}) &= \Pr\Big(\big(\text{Assigned to Heads} \wedge \text{Heads occurs}\big) \vee \big(\text{Assigned to Tails} \wedge \text{Tails occurs}\big)\Big)\\ &= \tfrac{1}{2}\cdot p + \tfrac{1}{2}\cdot(1-p) = \tfrac{1}{2}.\end{aligned}$$

Given this, I venture that the following beliefs are widely shared:

1. In an unfair decision procedure, the aggrieved has the right to demand an appeal, using a fair decision procedure, if she loses.

2. In an unfair decision procedure, the loser does *not* have the right to demand an appeal, using a fair decision procedure, if she was favoured.

Here the thought is that, only after the winner was determined, was it discovered that the decision procedure used was unfair. The initial plausibility of these beliefs would likely vary based on the specific problem under consideration, but if we think of the problem of assigning a kidney to one of two people for transplant, I think both beliefs strike us as plausible. I want to point out that these beliefs, although widely shared, yield an overall bias benefiting the *aggrieved*. And this requires us to adopt methods for the redress of grievances which might, initially, strike us as *unfair*.

Figure 1 illustrates a stereotypical decision tree for an unfair decision procedure. Since the first unfair decision procedure is independent of the second fair decision procedure, calculating the overall probability of the aggrieved player winning is trivial:

$$\begin{aligned}\Pr(\text{Aggrieved Wins}) &= \tfrac{1}{3} + \tfrac{2}{3} \cdot \tfrac{1}{2} \\ &= \tfrac{2}{3}.\end{aligned}$$

The reason why this problem warrants attention is that the two natural responses which solve the problem are curiously counterintuitive. The first response violates our intuitions about who has the right to appeal. The second response appears to compound the problem further by requiring the aggrieved to participate in another unfair decision procedure. Let us consider these in turn.

The first response denies condition 2 above. That is, any loser in an unfair decision procedure *does* have the right to demand an appeal, using a fair decision process. Doing so would restore equality to the overall probability of winning,

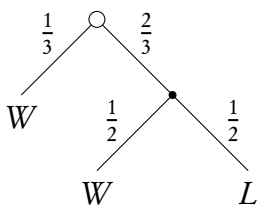
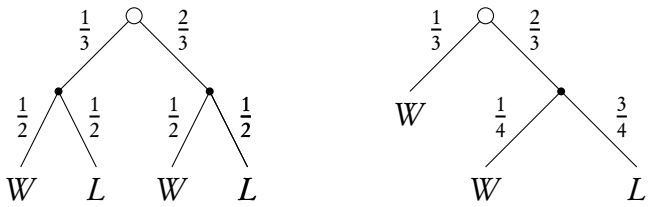


Figure 1: The decision tree for a stereotypical unfair decision procedure. Final status indicated is that of the aggrieved player.



(a) First response. (b) Second response.

Figure 2: Two possible responses to the puzzle of redressing grievances, both of which are counterintuitive. Final status indicated is that of the aggrieved player. Notice that, in both cases, the probability of both the aggrieved player and the favoured player winning equal $\frac{1}{2}$.

rendering it equally likely for either the aggrieved or favoured party. Figure 2(a) illustrates this. But notice the counterintuitive aspect of this response: suppose the initial probabilities were such that the favoured had a 99% chance of winning, but didn't. It's hard not to feel that the aggrieved is being hard-done by in being forced to relinquish the initial win and face another contest as a result of the favoured's request.

The second response agrees with conditions 1 and 2 above, in the sense of whom is allowed to appeal, but uses an unfair decision procedure for the second trial, with the *same* person being the aggrieved![2] Figure 2(b) illustrates this solution. The counterintuitive aspect of this response is that, in the second trial, the aggrieved player is again disadvantaged.

This problem turns on a simple arithmetic observation. But it is an important one because it calls attention to an unfortunate interplay regarding our intuitions about justice and fairness, expectations and outcomes. It is natural to think — when we are talking about perfectly symmetric persons, with no differences in need, ability, prior claims, and so on — that a just decision procedure is a fair decision procedure. And that, when an injustice occurs because an unfair decision procedure was used, the redress will always feature a fair decision procedure. But sometimes this is not the case. Sometimes the correct response to an injustice generated by an unfair decision procedure is to use *another* unfair decision procedure, which appears to disadvantage (in some sense) the same person again. In these cases, two wrongs do make a right.

[2] I would like to thank Richard Bradley for calling this second response to my attention.